Name:_____ Date:_____

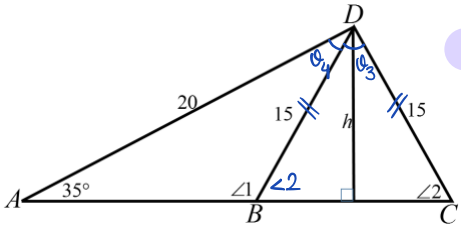
Pre-Calculus 11: HW 2.3b Ambiguous Case of Sine Law

1. Given each equation, solve for all values of θ where $0 \le \theta \le 360^\circ$. Note: There are two angles!!

a) $\sin\theta = \frac{2}{3}$ $\theta = \sin^{-1}\left(\frac{2}{3}\right)$ $\theta_1 = 41.8^\circ$ $\theta_2 = 138.2^\circ$	b) $\sin\theta = \frac{4}{5}$ $\theta = \sin^{-1}\left(\frac{4}{5}\right)$ $\theta_1 = 53.1^\circ$ $\theta_2 = 126.9^\circ$	c) $\sin\theta = -0.55$ $\theta = \sin^{-1}(-0.55) = -33.4$ ↓ Reference angle (Quadrant 4) $\theta_1 = 33.4^\circ$ $\theta_2 = 146.6^\circ$
d) $\sin\theta = \frac{-\sqrt{2}}{2}$ $\theta = \sin^{-1}\left(\frac{-\sqrt{2}}{2}\right)$ $\theta_1 = 45^\circ$ $\theta_2 = 135^\circ$	e) $\sin\theta = \frac{-\sqrt{3}}{2}$ $\theta = \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$ $\theta_1 = 60$ $\theta_2 = 120$	f) $\sin\theta = \frac{4}{\sqrt{7}}$ $\theta = \sin^{-1}\left(\frac{4}{\sqrt{7}}\right) \rightarrow \sin\theta = 1.5$ but $-1 \le \sin\theta \le 1$ so No answer $\theta_1 =$ ____ $\theta_2 =$ ____

2. Given each triangle, find the missing values and show all your work

a) Find the value of $\angle 1$, $\angle 2$, h, BC, *and* AB



$h \rightarrow \frac{20}{\sin 90^\circ} = \frac{h}{\sin 35^\circ}$

$h = \frac{20 \times (\sin 35^\circ)}{\sin 90^\circ}$

$h = 11.47$

$BC \rightarrow \theta_3 = 180 - 49.886 - 49.886$

$\theta_3 = 80.228^\circ$

$\frac{15}{\sin 49.886^\circ} = \frac{BC}{\sin 80.228^\circ}$ $BC = 19.33$

$\angle 1 \rightarrow \frac{15}{\sin 35^\circ} = \frac{20}{\sin\theta_1}$

$\angle 1 = \frac{\sin 35^\circ \times 20}{15} = \sin\theta_1$

$\sin\theta_1 = 0.765$ $\theta_1 = 180^\circ - 49.886^\circ$

$= 130.114^\circ$

$\angle 2 \rightarrow \frac{15}{\sin 35^\circ} = \frac{20}{\sin\theta_2}$

$\frac{20 \times \sin 35^\circ}{15} = \sin\theta_2$

$\theta_2 = 49.886^\circ$

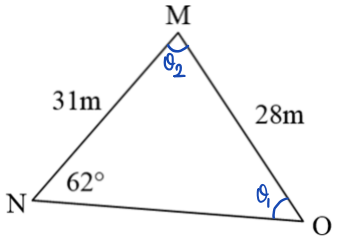
$AB \rightarrow \theta_4 = 180 - 35 - 130.114$

$\theta_4 = 14.886^\circ$

$\frac{15}{\sin 35^\circ} = \frac{AB}{\sin 14.886^\circ}$ $AB = 6.72$

$\angle 1 = 130.114^\circ$ $\angle 2 = 49.886^\circ$ $h = 11.47$ $BC = 19.33$ $AB = 6.72$

b) Find the value of $\angle MON, \angle OMN,$ and $\overline{ON}$



$\frac{28}{\sin 62^\circ} = \frac{31}{\sin \theta_1} \rightarrow \theta_1 = 77.84^\circ$

$\theta_2 = 180^\circ - 62^\circ - 77.84^\circ = 40.16^\circ$

$\frac{28}{\sin 62^\circ} = \frac{ON}{\sin 40.16^\circ}$

$ON = 20.45$

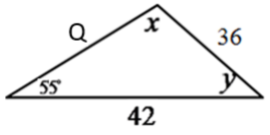
$\frac{28}{\sin 62^\circ} = \frac{ON}{\sin 15.84}$

$ON = 8.66$

$\angle MON =$ 77.84° (ACUTE) $\angle OMN =$ 40.16° $ON =$ 20.4

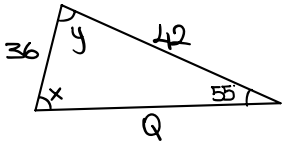
$\angle MON =$ 102.16° (OBTUSE) $\angle OMN =$ 15.84° $ON =$ 8.66

e)



$\angle X \rightarrow \frac{36}{\sin 55^\circ} = \frac{42}{\sin x} \rightarrow x = 72.877^\circ$

$\angle y \rightarrow 180 - 55 - 72.877 = 52.123^\circ$



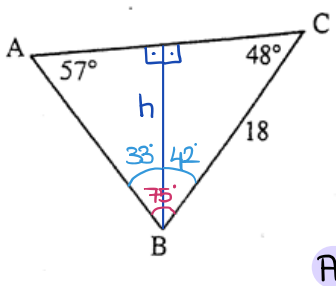
$Q \rightarrow \frac{36}{\sin 55^\circ} = \frac{Q}{\sin 52.123} \rightarrow Q = 34.689$

$Q \rightarrow \frac{36}{\sin 55^\circ} = \frac{Q}{\sin 17.877^\circ} \rightarrow Q = 13.191$

$\angle x =$ 72.877° (ACUTE) $\angle y =$ 52.123° $Q =$ 34.689

$\angle x =$ 107.123° (OBTUSE) $\angle y =$ 17.877° $Q =$ 13.191

3. Find the area of the following triangle. Note the area of a triangle is $A = b \times h \times 0.5$:



$\angle B = 180 - 48 - 57 = 75^\circ$

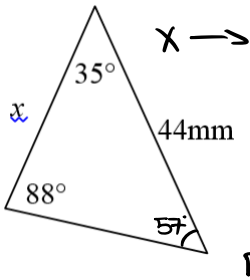
$|AC| = \frac{18}{\sin 57^\circ} = \frac{AC}{\sin 75^\circ} \rightarrow |AC| = 20.731$

$h = \frac{18}{\sin 90^\circ} = \frac{h}{\sin 48^\circ} \rightarrow h = 13.377$

Area $= 20.731 \times 13.377 \times 0.5 = 138.659\ cm^2$

4. Given each of the following triangles, indicate whether if there would be an ambiguous case. State the reason why or why not: Solve for "x".

a)

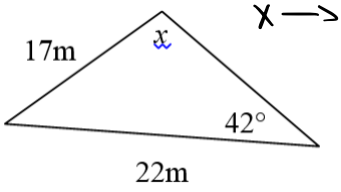


$x \rightarrow \frac{44}{\sin 88^\circ} = \frac{x}{\sin 57^\circ}$

$x = 36.92$

no ambiguous case bc it is not side-side-angle

b)

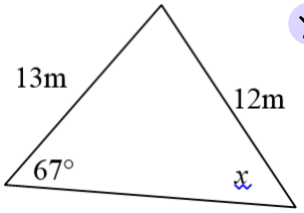


$x \rightarrow \frac{17}{\sin 42^\circ} = \frac{22}{\sin x^\circ}$

$x = 59.99^\circ$

yes, there is ambiguous case bc opposite side of the given angle is smaller than the other side.

c)

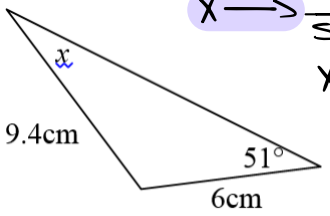


$x \rightarrow \frac{12}{\sin 67^\circ} = \frac{13}{\sin x^\circ}$

$x = 85.722^\circ$

yes, there is ambiguous case bc opposite side of the given angle is smaller than the other side.

d)

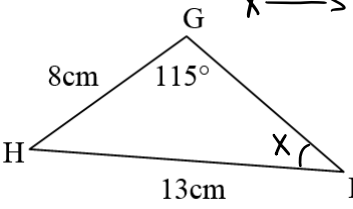


$x \rightarrow \frac{9.4}{\sin 51^\circ} = \frac{6}{\sin x^\circ}$

$x = 29.739^\circ$

no it isn't ambiguous case bc opposite side of the given angle is bigger than the other side.

e)

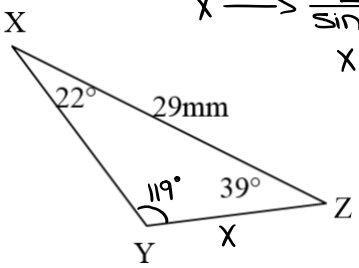


$x \rightarrow \frac{13}{\sin 115^\circ} = \frac{8}{\sin x}$

$x = 33.899^\circ$

no it isn't ambiguous case bc opposite side of the given angle is bigger than the other side.

f)



$x \rightarrow \frac{29}{\sin 119^\circ} = \frac{x}{\sin 22^\circ}$

$x = 12.421$

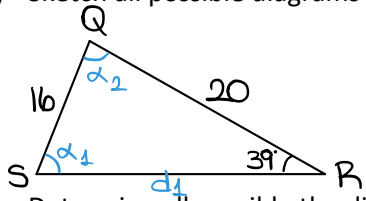
no ambiguous case bc it is not side-side-angle

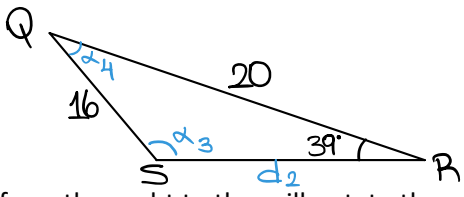
5. A lighthouse at point Q is 20 km from a yacht at point R and 16 km from a sailboat at point S. From the yacht, the lighthouse and the sailboat are separated by an angle of $39°$

a) Is it necessary to consider the ambiguous case? Explain.

yes, there is ambiguous case bc opposite side of the given angle is smaller than the other side.

b) Sketch all possible diagrams for this situation.





c) Determine all possible the distances from the yacht to the sailboat, to the nearest tenth of a kilometre.

$\alpha_1 \rightarrow \frac{16}{\sin 39°} = \frac{20}{\sin \alpha_1}$

$\alpha_1 = 51.874°$

$\alpha_2 \rightarrow 180 - 39 - 51.874$

$\alpha_2 = 89.126°$

$d_1 \rightarrow \frac{16}{\sin 39°} = \frac{d}{\sin 89.126}$

$d = 25.421$ km

$\alpha_3 \rightarrow 180 - 51.874 = 128.126°$

$\alpha_4 \rightarrow 180 - 39 - 128.126 = 12.874°$

$d_2 \rightarrow \frac{16}{\sin 39°} = \frac{d}{\sin 12.874}$

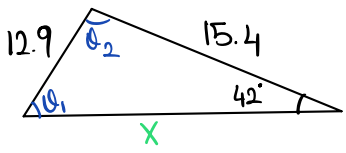
$d = 5.665$ km

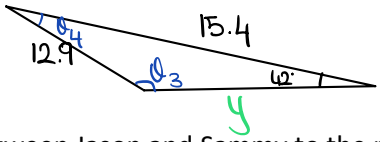
6. Jason and Sammy are part of a scientific team studying clouds. The team is about to launch a weather balloon into an active part of the cloud. Jason's rope is 15.4 m long and makes an angle of $42°$ with the ground. Belle's rope is 12.9 m long.

a) Is it necessary to consider the ambiguous case? Explain.

Yes it is because both triangles meet the expectations of the question.

b) Sketch all possible diagrams for this situation.





c) Determine all possible the distances between Jason and Sammy to the nearest tenth of a meter.

$\frac{12.9}{\sin 42°} = \frac{15.4}{\sin \theta_1}$

$\theta_1 = 53.02°$

$\theta_2 = 84.98°$

$\frac{12.9}{\sin 42°} = \frac{x}{\sin 84.98°}$

$x = 19.20$ m

$\theta_3 = 180° - 53.01° = 120.99°$

$\theta_4 = 180° - 42° - 120.99° = 17.1°$

$\frac{12.9}{\sin 42°} = \frac{y}{\sin 17°}$

$y = 5.64$ m